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Effective Conductivity Concept for Modeling Conductor Surface Roughness

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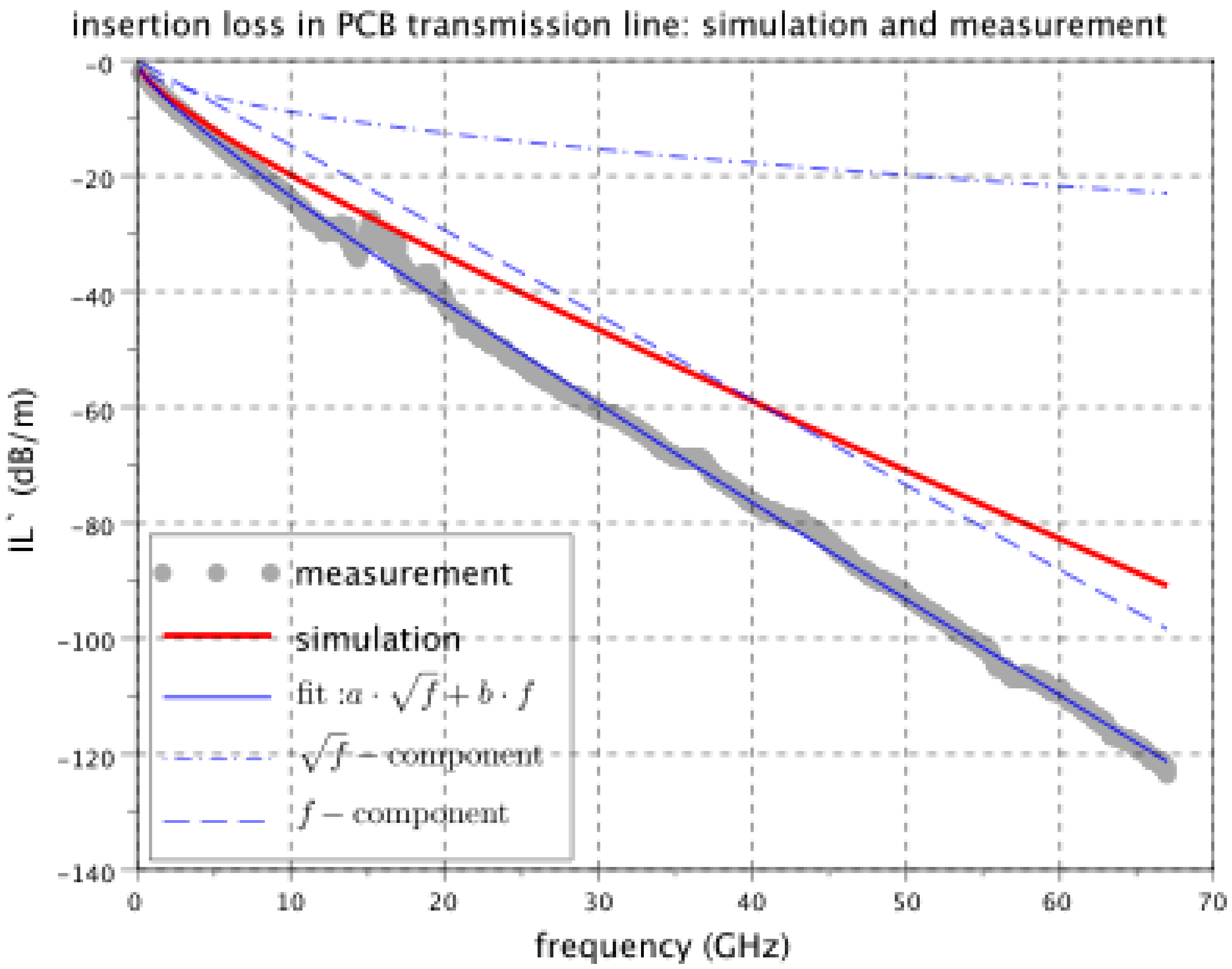
Contents

Introduction

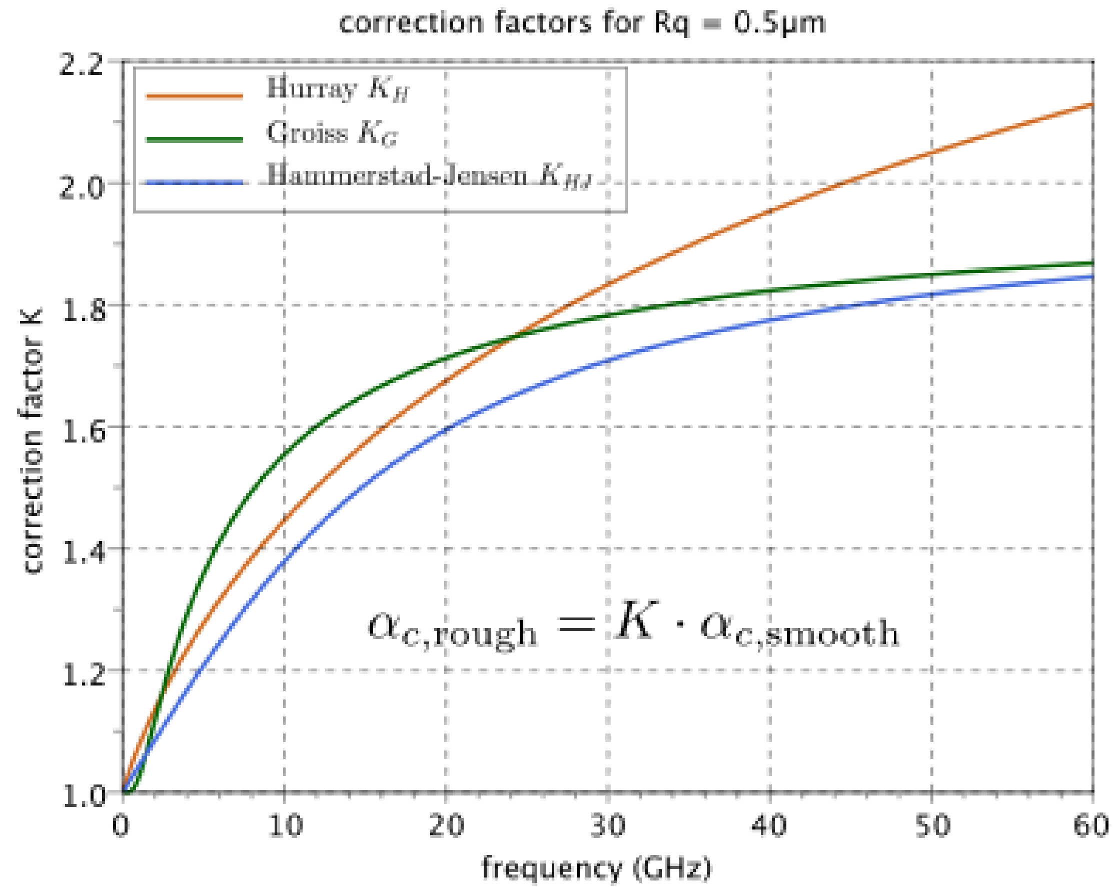
- Accurate models of conductor behavior are essential for predictive simulations: antennas, resonators, connectors, filters, transmission lines
- Today, PCB transmission lines are operated up to the two digit Gigahertz range

- **Transmission line loss:**
- Usally dominated by dielectric loss $\propto f$
- Theory predicts a $\sqrt{f}$ - dependence due to skin effect for conductor loss

- **Skin effect at frequencies > 1GHz:**
- Skin depth decreases to the order of surface roughness:
  - conductor surface can no longer be regarded as ideally smooth
  - $\sqrt{f}$- dependence is no longer valid



Existing Models for Surface Roughness



- **Phenomenological models:**
- Correction factor K adapted to measurement
- Function of RMS roughness R_q

• Hammerstad & Jensen $$K_{HJ} = 1 + \frac{2}{\pi}\tan^{-1}\left(1.4\,\frac{R_q}{\delta}\right)$$

• Groiss $$K_G = 1 + \exp\left(-\left(\frac{\delta}{2R_q}\right)^{1.6}\right)$$

• Fail at high frequencies resp. high values of R_q

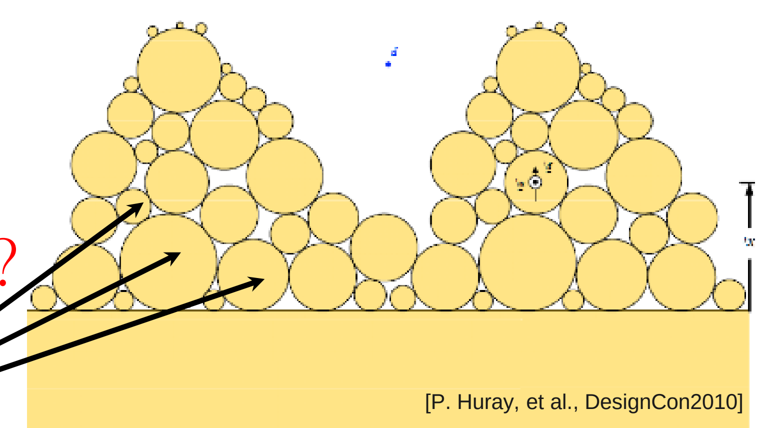
- **Physical models:**
- Huray's „snowball"-modell:

analytical calculation of additional power loss due to copper „snowballs"

$$K_H = 1 + \frac{3}{2}\sum_{i=1}^{j}\left(\frac{N_i}{A_{hex}\cdot 4\pi a_i^2}\right)\cdot\left(1+\frac{\delta}{a_i}+\frac{\delta^2}{a_i^2}\right)^{-1}$$

- **Others:**
- Fractal surface, „brute force" 3D simulation, …
- major drawbacks**:** many parameters, often hardly observable

N_i, a_i ?



Inconsistencies of Common Modeling Approaches

Current 'indirection'

- The displacement amplitude of an conduction electron is only ≈10^{-12}m (P= 10mW at 1GHz)
- There is no 'current indirection' in a rough surface in a sense that the current path follows any 'surface contour'

Model roughness as piecewise smooth facets

- Surface profiles cannot be modeled by ideally smooth facets because then the solver assumes the skin effect for an infinite plane on their surfaces
- The plane skin effect is only valid if feature sizes >> λ

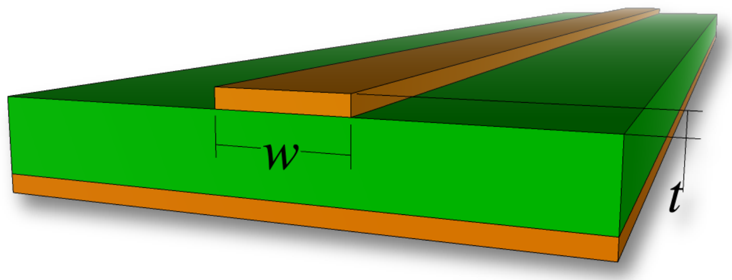
Model microscopic features

- It is not necessary to simulate surface roughness on a microscopic level because it is averaged over the order of a wavelength anyway

Dimensions Consideration

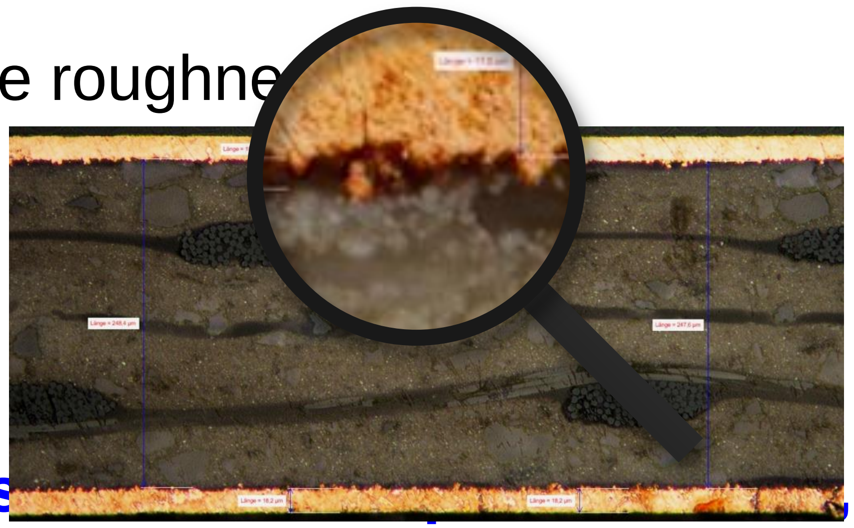
Relevant dimensions for $f \approx 1\text{–}100\,\mathrm{GHz}$:

- Wavelength: $\lambda \approx 2 - 200\,\mathrm{mm}$
1. Trace width: $w \approx 100\,\mu\mathrm{m}$
- Trace thickness: $t \approx 18\,\mu\mathrm{m}$
- Skin depth (in Cu): $\delta \approx 0.2 - 2\,\mu\mathrm{m}$



Situation on PCB in the operating frequency range:

- Wave length $\lambda \gg$ conductor dimensions w, t
- Conductor dimensions $w, t \gg$ skin depth δ Skin depth $\delta \lesssim$ surface roughne

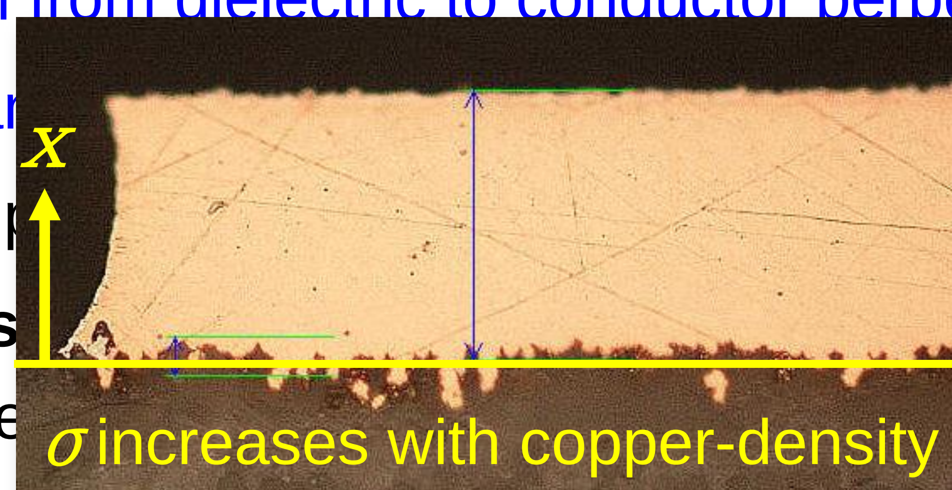


Conclusion:

- As $w, t \gg R_q$, the conductor surface basically is “plane”
- But there is roughness on a microscopic scale $(\ll \lambda, w, t)$
- **Propagating wave does not “see“ individual peaks and pits**
however with no abrupt border between dielectric and conductor

Modeling Approach

- No abrupt border between dielectric and conductor
- Not necessary to model **microscopic** peaks
- Rather model the transition from dielectric to conductor perpendicular to the surface
- Maintain translation invariar
- Appropriate **macroscopic** p s transition: conductivity σ
- **model surface roughness rpendicular to the surface:**
  - σ is a function of distance
  - $\sigma(x)$ is proportional to the probability of finding metal in a plane parallel to the surface
  - $\sigma(x)$ increases from virtually zero in the dielectric to bulk metal conductivity



Surface Roughness Characterization

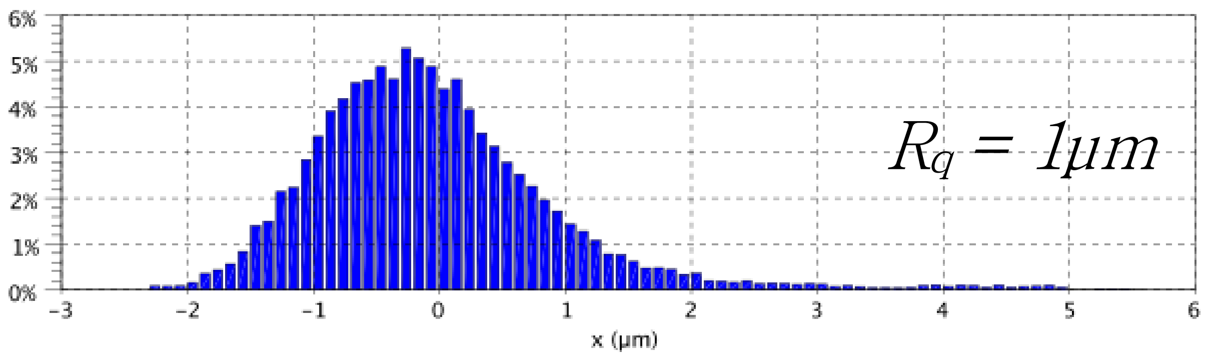
- Assume normally distributed surface profile
- $\sigma(x)$ corresponds to the cumulative distribution function (CDF) of surface profile:

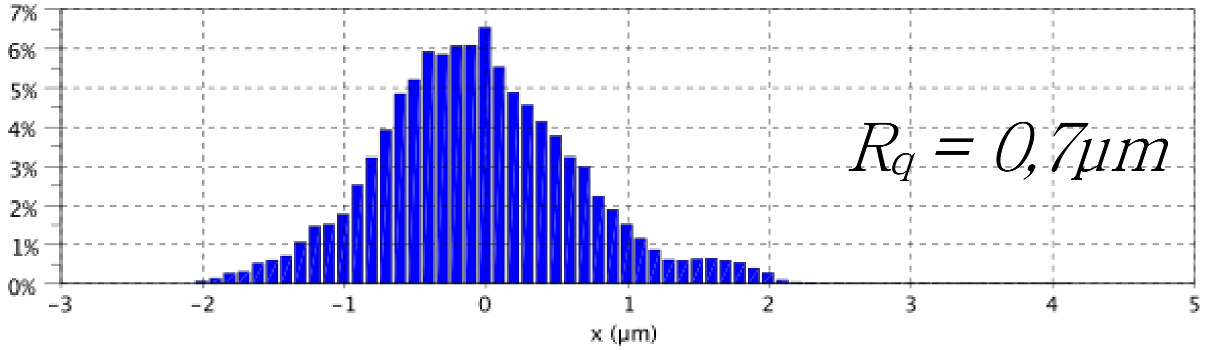
$$\sigma(x) = \sigma_{bulk} \cdot \int_{-\infty}^{x} PDF(x)\, du = \sigma_{bulk} \cdot \int_{-\infty}^{x} \exp\left(-\frac{u^2}{2R_q^2}\right) du$$

- **Surface characterisation:**
- Optical scanning system: Confirms normal distribution

➠ **One single model parameter !**

measured PDF





Conductivity Gradient Model Derivation

Skin effect in rough surfaces is deduced from Maxwell's equations with time harmonic fields and location dependent conductivity $\sigma(r)$:

Using Maxwell-Ampere's Law: $\underline{\nabla} \times \left(\frac{\underline{B}}{\mu_r} \right) = j\omega \frac{1}{c^2} \left(\varepsilon_r \underline{E} \right) + \mu_0 \underline{J}$, inserting Ohm's Law $\underline{J} = \sigma \underline{E}$

for $\mu_r = 1$ yields: $\underline{\nabla} \times \underline{B} = \left(j\omega \frac{\varepsilon_r}{c^2} + \mu_0 \sigma \right) \underline{E}$

Even at $f = 100$GHz, $\omega \varepsilon_r / c^2 \ll \mu_0 \sigma$, if $\sigma \gg 5.6$S/m, so displacement current density can be neglected for σ down to ≈1ppm of copper conductivity:

$$\underline{\nabla} \times \underline{B} = \mu_0 \sigma \underline{E}$$

Taking the curl, $\underline{\nabla}\times$: $\underline{\nabla} \times \underline{\nabla} \times \underline{B} = \mu_0 \underline{\nabla} \times \left(\sigma \underline{E} \right)$

$\underline{\nabla} \times \underline{\nabla} \times \underline{A} = \underline{\nabla}\left(\underline{\nabla} \cdot \underline{A} \right) - \Delta \underline{A}$ ↙ ↙ $\underline{\nabla} \times \left(\varphi \underline{A} \right) = \varphi \underline{\nabla} \times \underline{A} - \underline{A} \times \underline{\nabla} \varphi$

we get $-\Delta \underline{B} = \mu_0 \sigma \underline{\nabla} \times \underline{E} - \mu_0 \underline{E} \times \underline{\nabla} \sigma$

which, with Faraday's Law $\underline{\nabla} \times \underline{E} = -j\omega \underline{B}$ ↓ $\underline{E} = \frac{1}{\mu_0 \sigma} \underline{\nabla} \times \underline{B}$

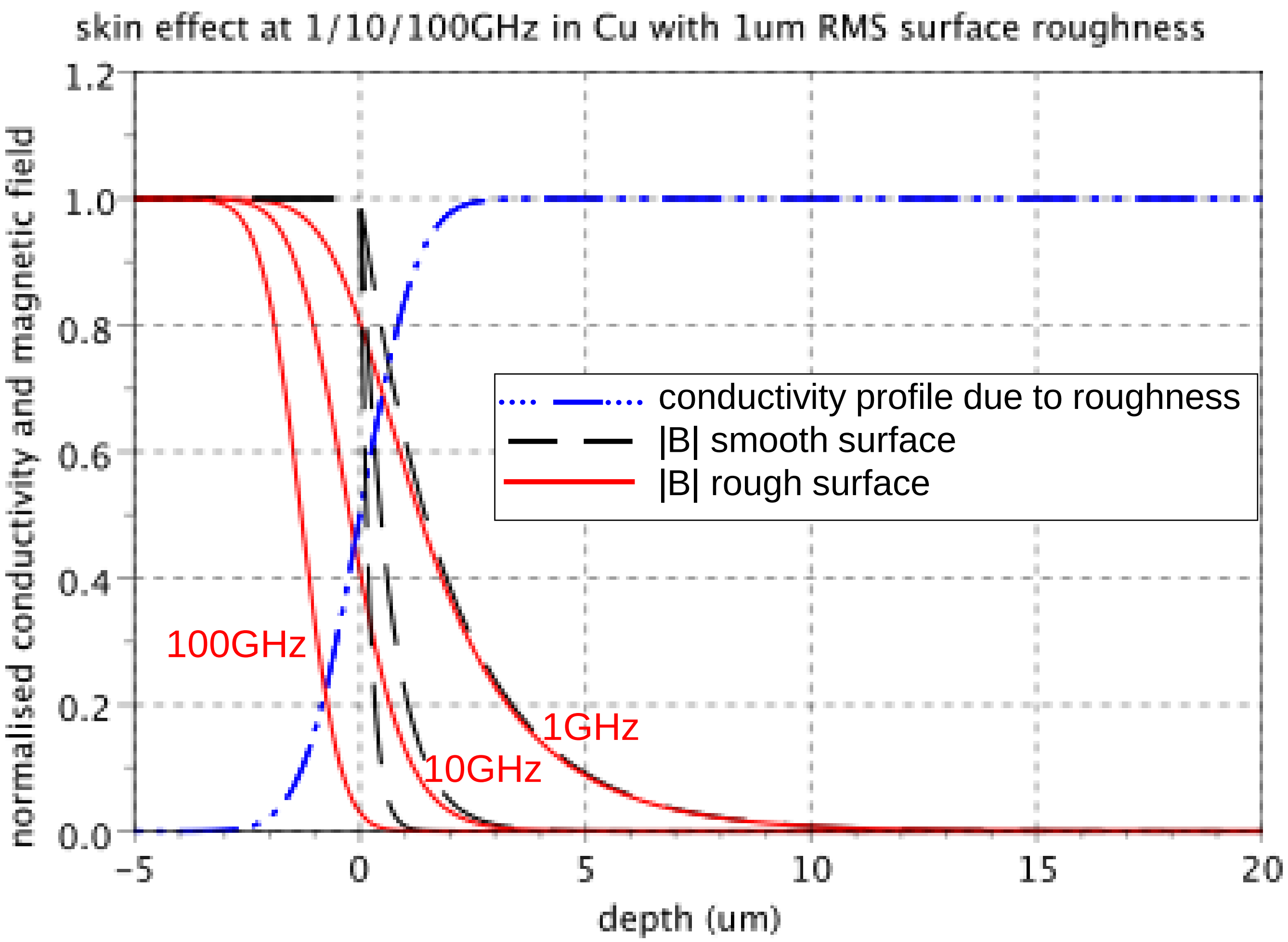
results in: → $\Delta \underline{B} - j\omega \mu_0 \sigma \underline{B} + \frac{\underline{\nabla} \sigma}{\sigma} \times \left(\underline{\nabla} \times \underline{B} \right) = 0$

Skin Effect in Rough Surfaces

- Assume gradual transition to bulk conductivity perpendicular to the mean surface
  - Focus on one-dimensional problem

- **The gradient model**
- Not only delivers a correction factor
- Describes skin-effect in rough surfaces as a whole by one parameter: RMS-roughness R_q
- Calculates profiles of magnetic field strength and loss power density for a given roughness distribution

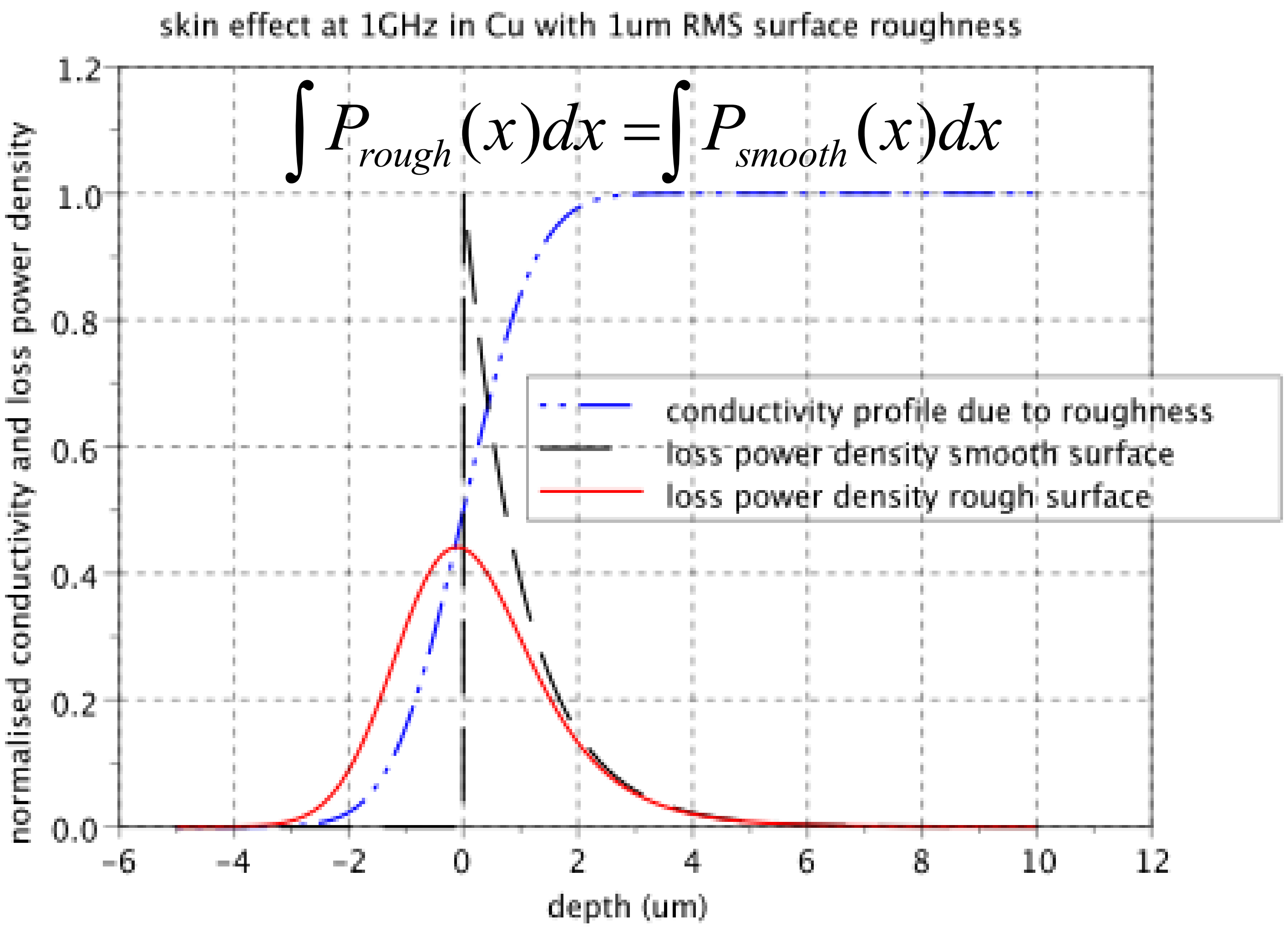
- **Skin effect in ideally smooth surface:**
- Magnetic field abruptly starts to decline



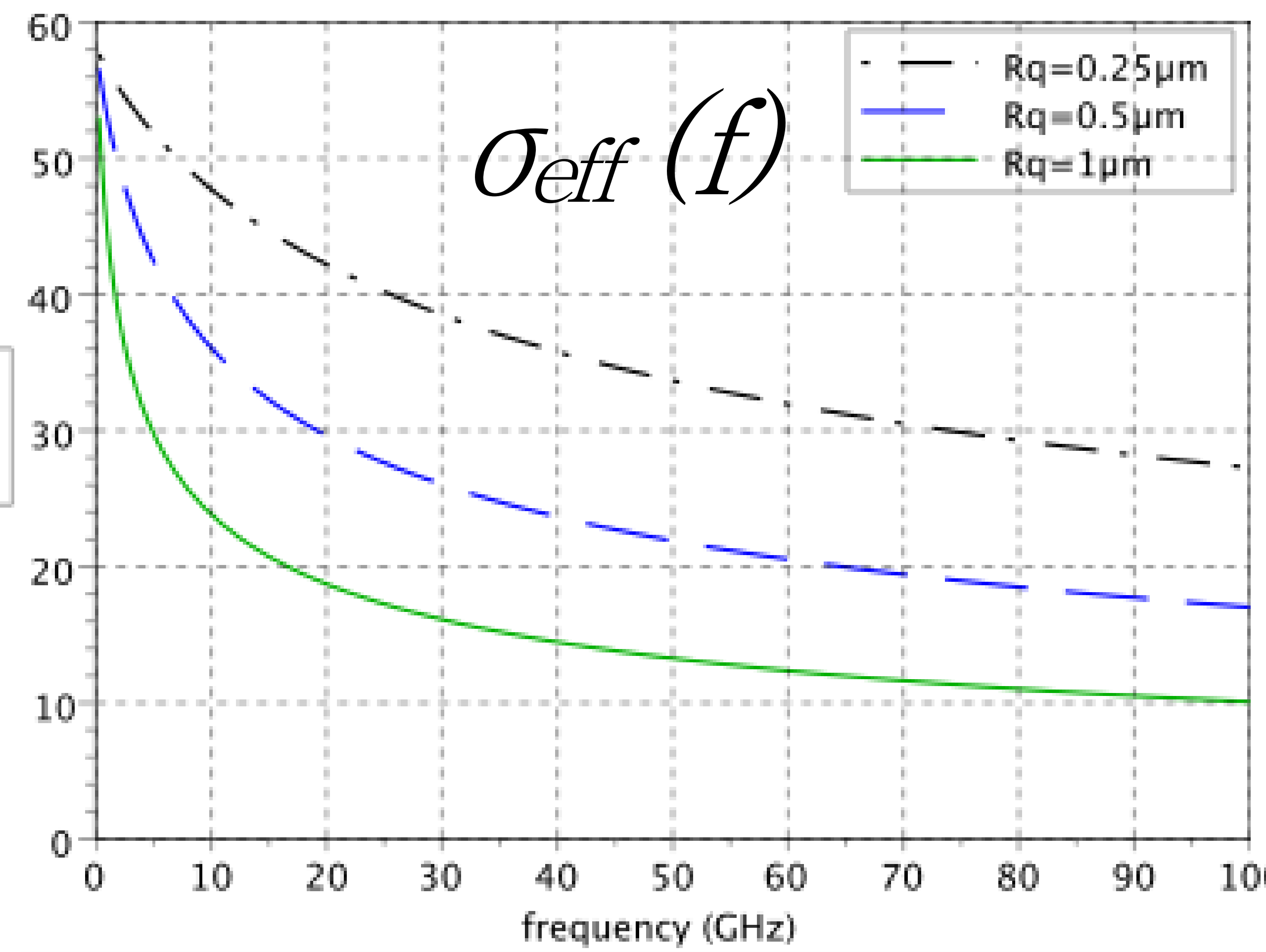
- **Gradient model for skin effect in rough surface:**
- Magnetic field smoothly decreases as it enters the range of surface roughness

Concept of Effective Conductivity

- Comparison of loss power densities: Gradient Model vs. conventional skin effect
- Effective conductivity is defined as the conductivity of a material with ideally smooth surface that would cause the same loss as the rough surface

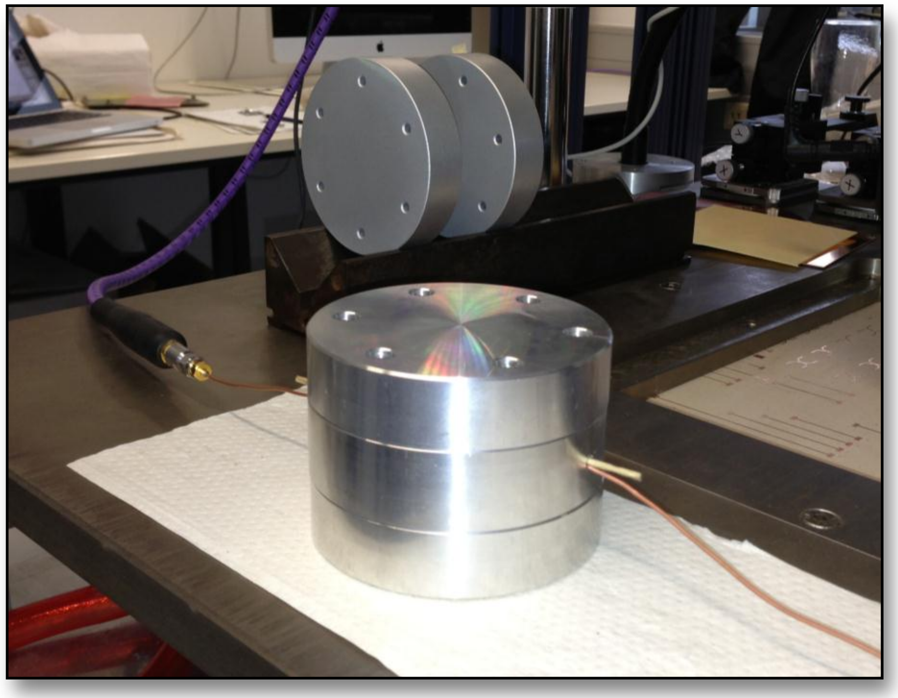


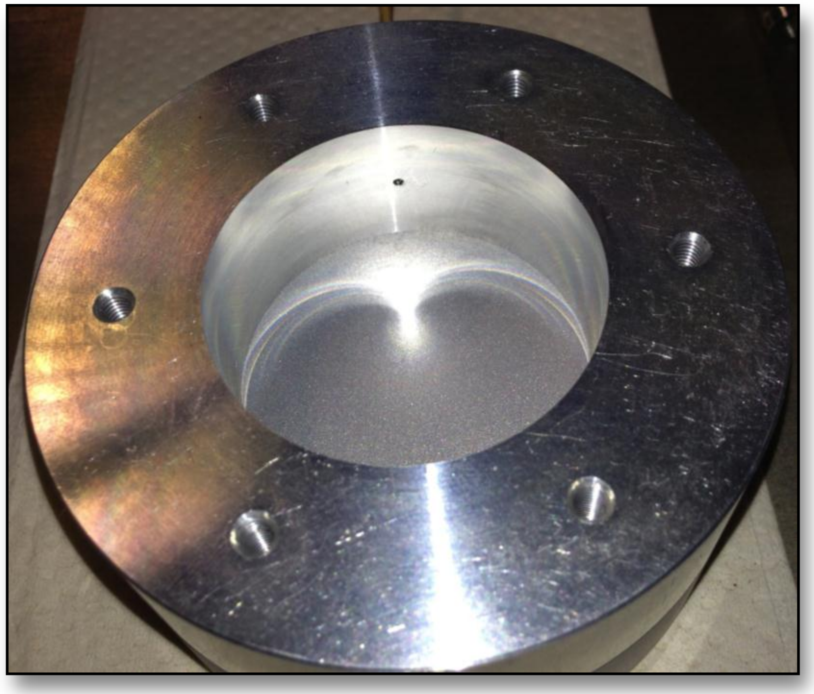
frequency dependent effective conductivity

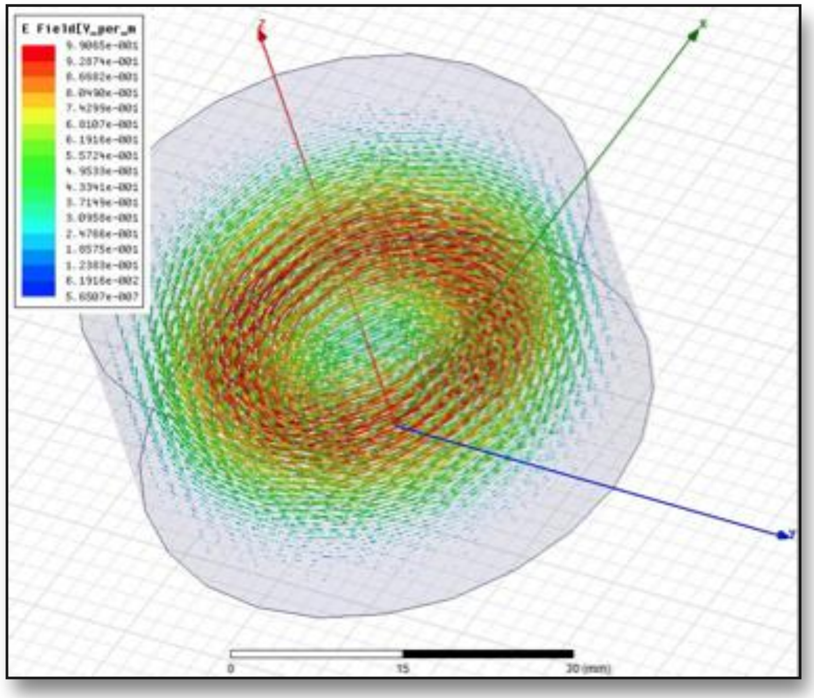


Direct Measurement of the Effective Conductivity

- Cylindrical cavity resonator operated at ≈ 10, 15, 21GHz (H_{01x} modes)
- Electrical field has circular component E_ϕ only
- Exchangeable lids of different surface roughness R_q
- Influence of surface roughness measurable by quality factor Q
- Treating rough surfaces as if they were ideally smooth yields σ_{eff}

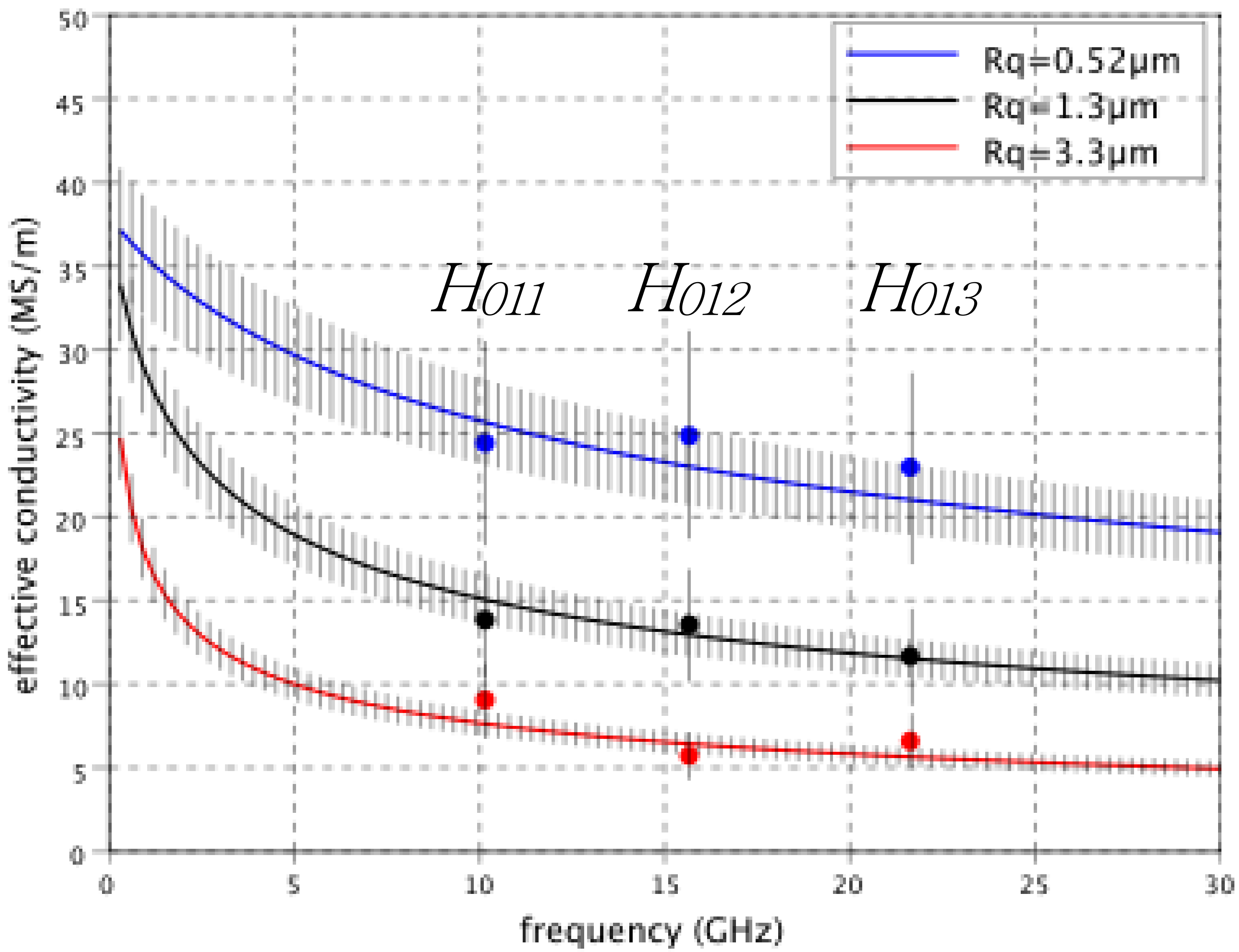






Direct Measurement of the Effective Conductivity

- RMS-roughness R_q of lids was obtained by optical scanning system
- Responses for σ_{eff} predicted by Gradient Model from R_q agree with measured σ_{eff}:

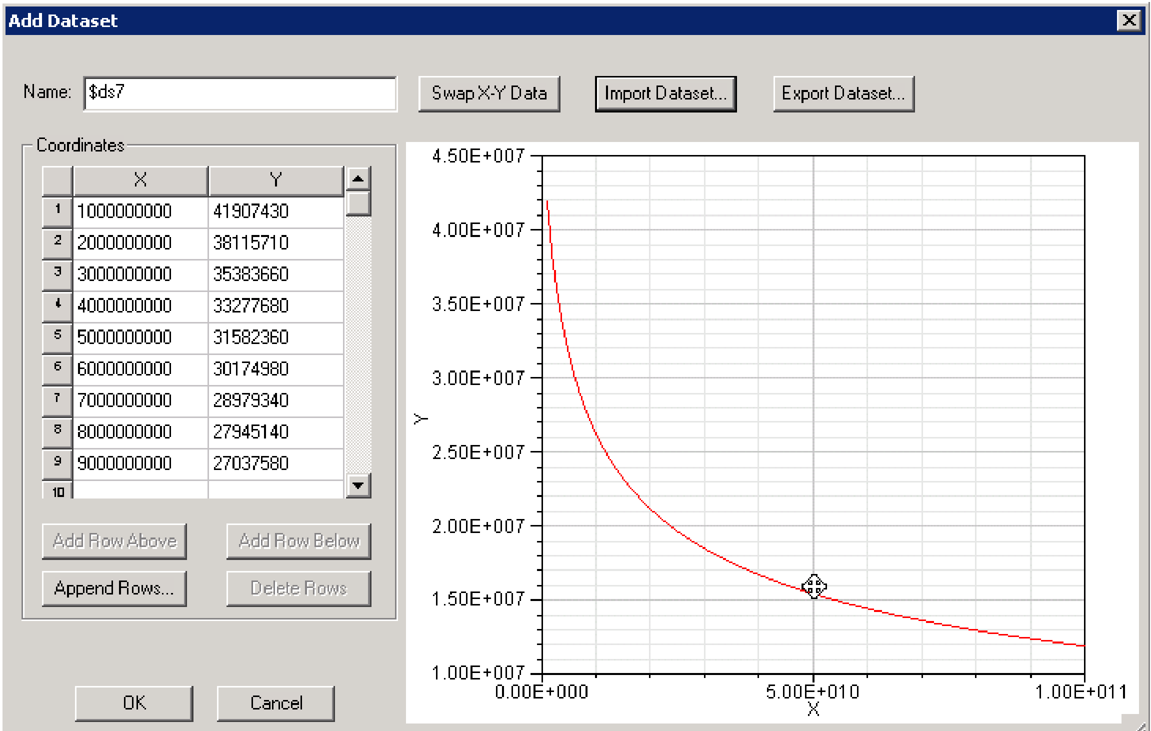


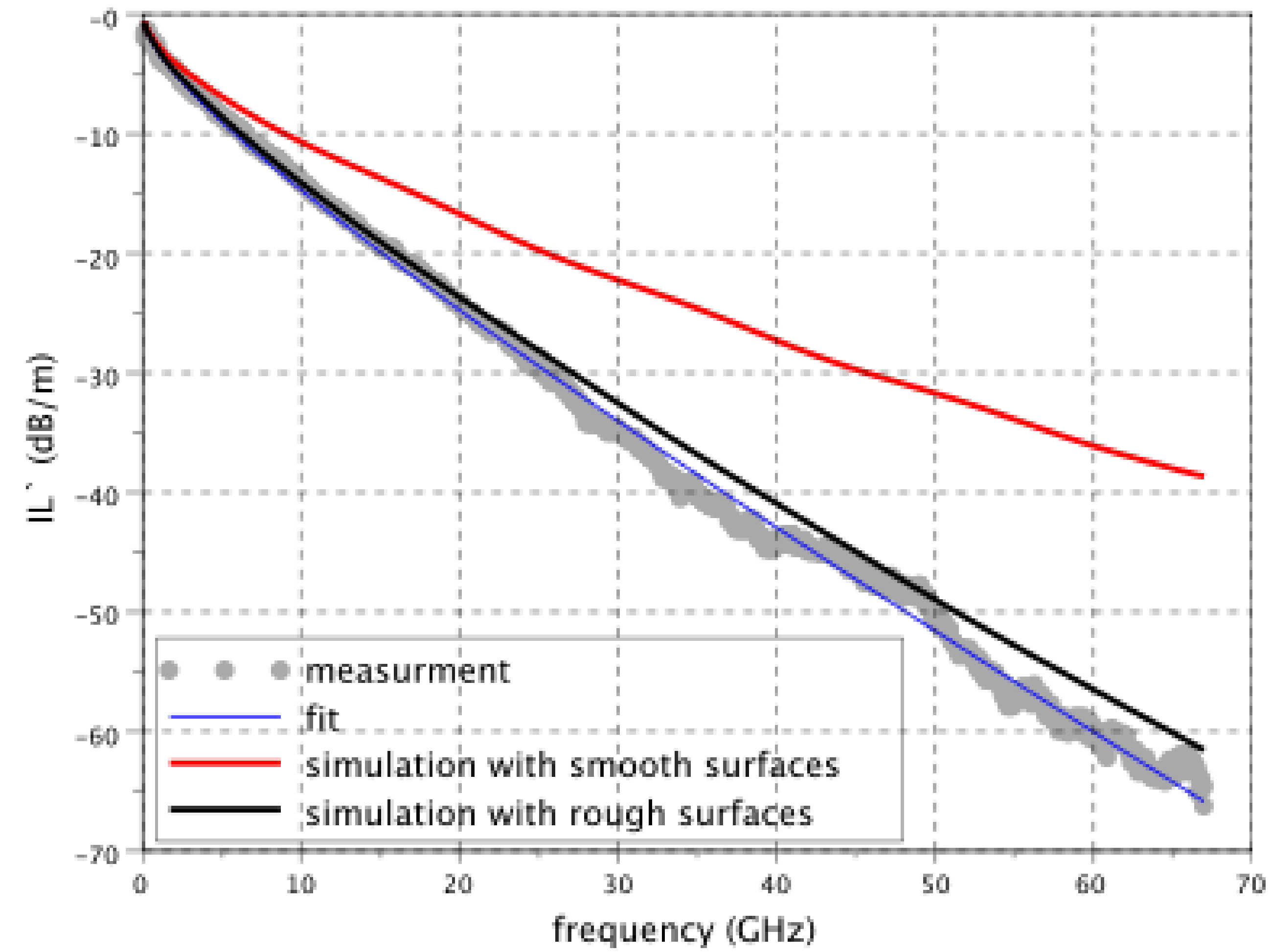
Application in Field Solvers as Impedance Boundary Condition

Advantages of impedance boundary conditions:

- Not necessary to mesh inside the conductor
- No increase of computation time
- Impedance boundary condition with $Z_s = \frac{1+i}{\sigma\delta}$ projects field phenomena in the conductor to its surface

➡Modification: $\sigma \to \sigma_{eff}(f)$, $\delta \to \delta(\sigma_{eff})$!





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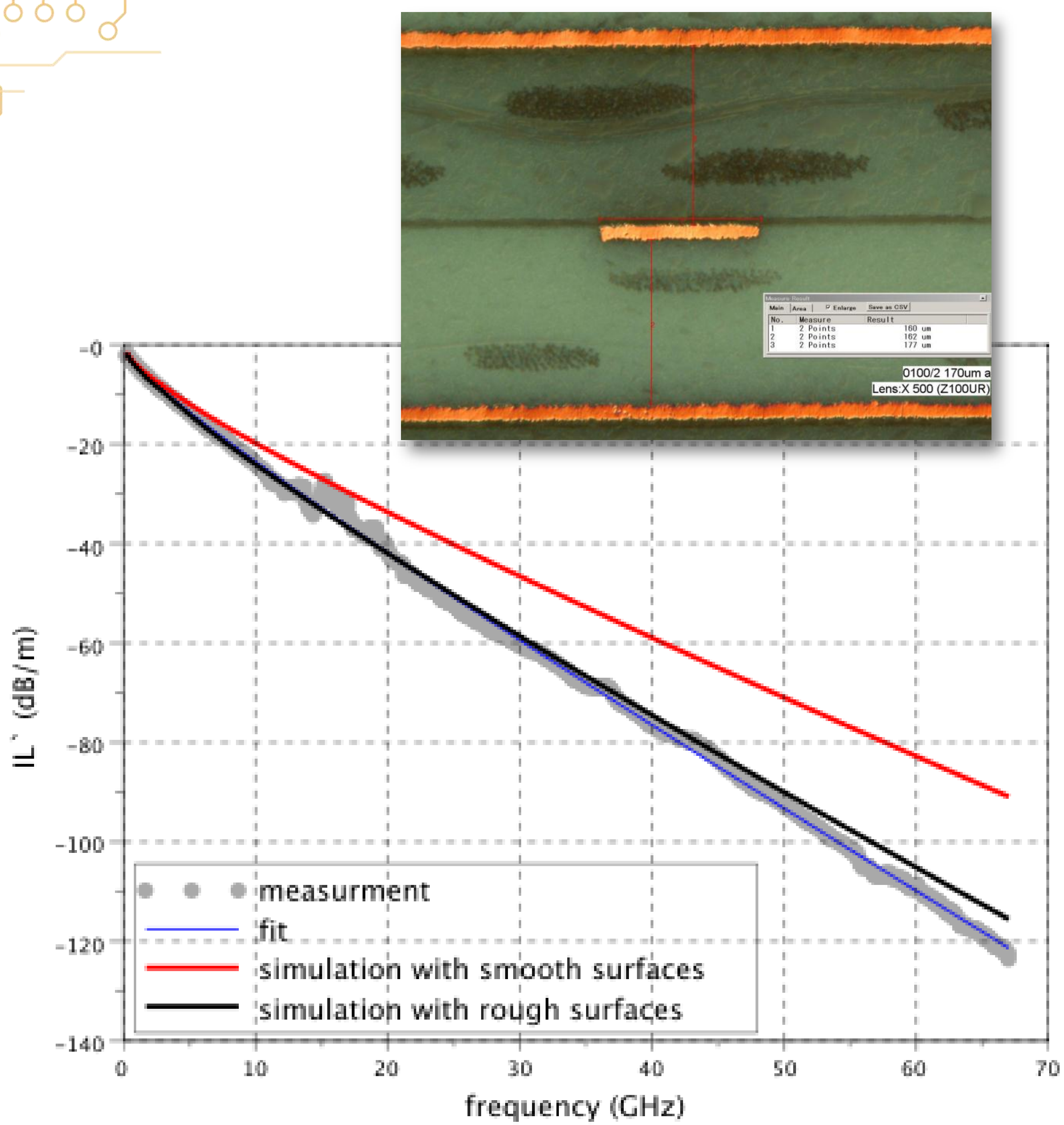

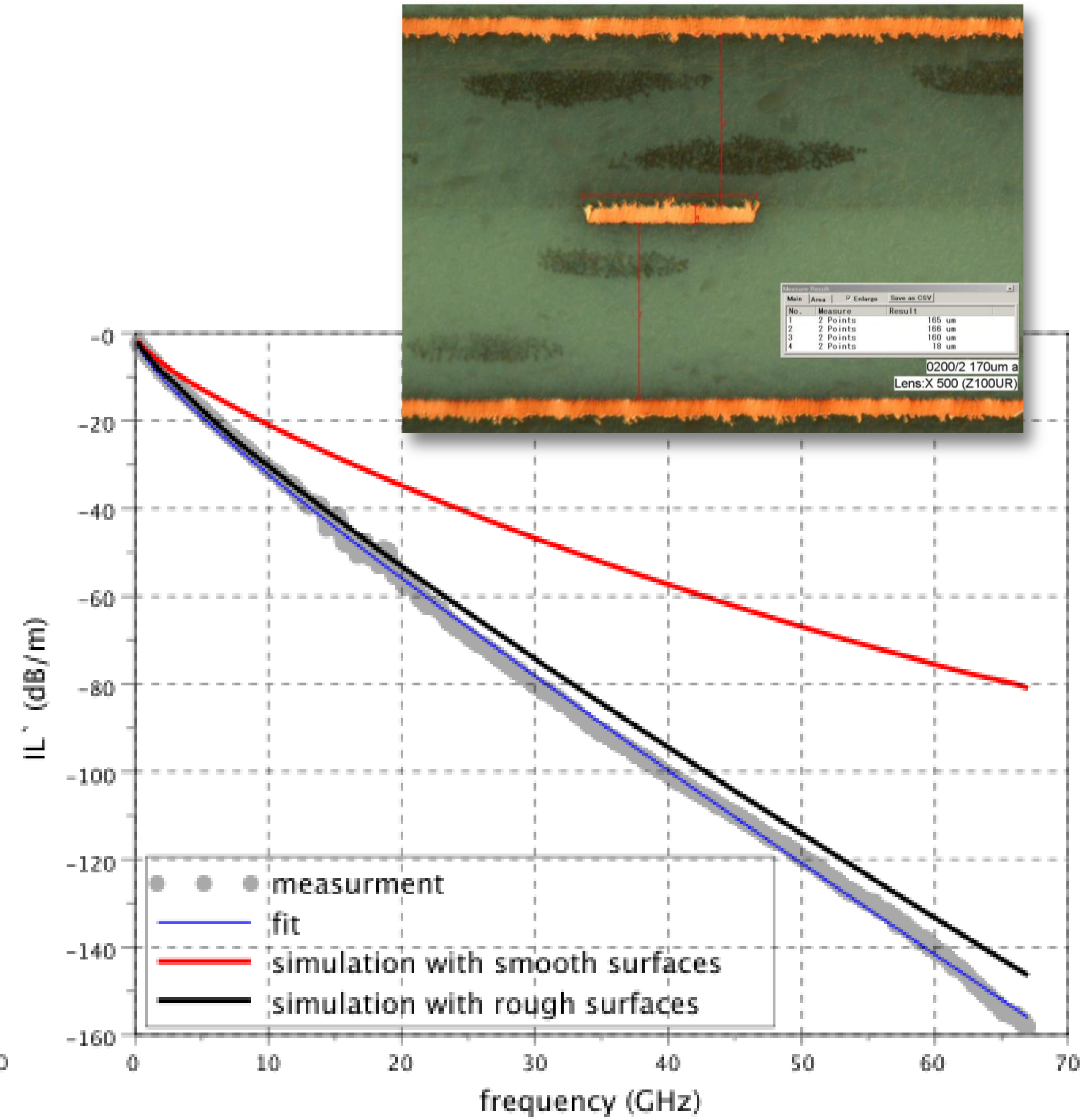

Conclusion

- Surface roughness is modeled as a conductivity gradient $\sigma(x)$
  - $\sigma(x)$ is proportional to the CDF of the surface roughness profile
  - Single parameter model: R_q
  - R_q measurable with optical scanning system or microsection
  - Datasheets often provide values for R_q

- Frequency dependent effective conductivity σ_{eff}
  - Derived from comparison to loss power density of smooth surface
  - Surface impedance as boundary condition
  - Easily applicable with commercial field solvers: import once to library
  - No increase of computation time