Accurate Inductance Modeling of 3-D Inductor Based on TSV

Shilong Gou, Gang Dong, Zheng Mei, and Yintang Yang

Abstract—**In this letter, an accurate model for the inductance of 3-D integrated inductor based on through silicon via is proposed. The model, considering both the internal inductance and detailed calculation of redistribution layer inductance, can more precisely obtain the total inductance of 3-D inductor using negligible computational time. Compared with the results of the Q3D extractor, the inductance results obtained from the proposed model exhibit good agreement with various design parameter ranges. The maximum error is less than 3.5%, so our model achieves high accuracy.**

Index Terms—**3-D integrated inductor, inductance model, redistribution layers (RDL), through silicon via (TSV).**

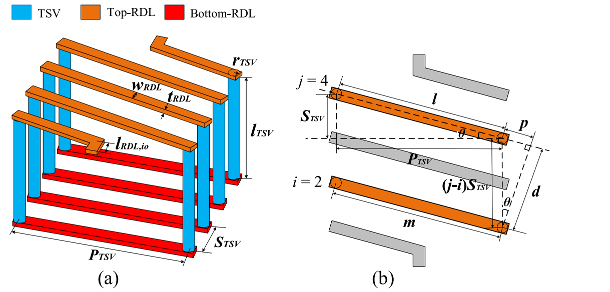

Fig. 1. (a) Structure and design parameters of TSV-based 3-D inductor (four turns). (b) Top view of top-RDL of the inductor.

I. Introduction

As a critical enabling technique for 3-D integrated circuits, the through silicon via (TSV) can provide high aspect ratio interconnection for multilayer chips in the vertical direction to realize the integration of heterogeneous chips [1]. Moreover, TSVs with redistribution layers (RDLs) are also developed in multiple applications such as various integrated passive devices [2]. Thus, it is a new choice to realize 3-D on-chip inductors using TSVs and RDLs. Compared with traditional planar inductors, the TSV-based inductors can occupy less die area and get a higher inductance density by making use of the vertical direction.

At present, considerable research has been conducted on the TSV-based 3-D inductor. However, the inductance value of the inductor is obtained by the empirically approximated expressions based on Y-parameters from the 3-D full-wave simulation in most works [3], which requires a lot of time and computational resources. Hence, the analytical method is required to calculate the inductance value. Wang and Yu [4] proposed a simple inductance model of 3-D inductor according to the self and mutual inductance of TSVs and RDLs. Due to many approximations are used in this method, large errors are introduced, especially when the size of the inductor is large.

In Section II, we developed a precise inductance model based on geometries for the total inductance of the inductor.

II. Inductance Model Description

Referred to the implemented 3-D inductors [3], the physical structure of TSV-based 3-D inductor (four turns) is shown in Fig. 1(a), where l_{TSV} and r_{TSV} denote the length and radius of TSV. The width and thickness of RDL are denoted by w_{RDL} and t_{RDL}. The conductor segments of the inductor can be divided into four groups: TSVs from the same row, TSVs from different rows, RDLs from the top layer (top-RDL), and the RDLs from the bottom layer (bottom-RDL). As shown in Fig. 1(a), P_{TSV} and S_{TSV} denote the distances between TSVs from different rows and the same row, respectively.

The partial self-inductance of an individual TSV of radius r and length of l is expressed as in (1), which is developed using the magnetic flux density [5]. Considering that a 3-D inductor of N turns contains $2N$ TSVs which are connected in series, the total self-inductance of TSVs can be derived from (2). Where μ is the permeability of the surrounding medium, here the surrounding materials of TSV are not ferromagnetic, so $\mu = \mu_0 = 4 \times 10^{-7}$ is permeability of the vacuum in this letter

$$L_{S1}(l,r) = \frac{\mu}{2\pi} l \left[\ln\left(\frac{l}{r} + \sqrt{\left(\frac{l}{r}\right)^2 + 1} \right) - \sqrt{\left(\frac{r}{l}\right)^2 + 1} + \frac{r}{l} + \frac{1}{4} \right] \quad (1)$$

$$L_{\mathrm{TSV},S} = 2N L_{S1}(l_{\mathrm{TSV}}, r_{\mathrm{TSV}}) \quad (2)$$

$$L_{S2}(l,a,b) = \frac{\mu}{2\pi} l \left[\ln\left(\frac{2l}{a+b}\right) + \frac{1}{2} + \frac{a+b}{3l} \right] + \frac{\mu l}{8\pi} \quad (3)$$

$$\begin{aligned} L_{\mathrm{RDL},S} = {} & N L_{S2}(l_{\mathrm{RDL},b}, w_{\mathrm{RDL}}, t_{\mathrm{RDL}}) \\ & + (N-1) L_{S2}(l_{\mathrm{RDL},t}, w_{\mathrm{RDL}}, t_{\mathrm{RDL}}) \\ & + 2L_{S2}(l_{\mathrm{RDL},t}/2, w_{\mathrm{RDL}}, t_{\mathrm{RDL}}) \\ & + 2L_{S2}(l_{\mathrm{RDL,io}}, w_{\mathrm{RDL}}, t_{\mathrm{RDL}}). \end{aligned} \quad (4)$$

The partial self-inductance of an individual RDL interconnect is expressed as in (3), where $\mu l/8\pi$ is internal inductance of the RDL. The internal inductance is considered in this formula because the magnetic fields are existed both inside and outside the conductor. Where l is the length of the RDL, a and b are dimensions of the rectangular cross section. There are several different lengths of RDLs in an N-turns 3-D inductor, included that N bottom-RDLs of length $l_{\mathrm{RDL},b} = P_{\mathrm{TSV}}$, $(N-1)$ top-RDLs of length $l_{\mathrm{RDL},t} = (P_{\mathrm{TSV}}^2 + S_{\mathrm{TSV}}^2)^{1/2}$, two top-RDLs of length $l_{\mathrm{RDL},t}/2$ and two top-RDLs used as input or output of signal (IO-RDL) of length $l_{\mathrm{RDL,io}}$, which

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is defined in Fig. 1(a). Similarly, the RDLs in inductor are connected in series, so the total self-inductance of RDLs can be determined by (4).

The partial mutual inductance between two TSVs can be calculated using (5), where l is the length of the TSV and d is the distance between the two TSVs. Thus, the total mutual inductance between TSVs from the same row in the 3-D inductor can be determined by (6), where $d_{ij} = (j-i)S_{\mathrm{TSV}}$ is the distance between the ith and jth TSVs from the same row

$$M_{p1}(l,d) = \frac{\mu}{2\pi} l \left[\ln\left(\frac{l}{d} + \sqrt{\left(\frac{l}{d}\right)^2 + 1} \right) - \sqrt{\left(\frac{d}{l}\right)^2 + 1} + \frac{d}{l} \right] \tag{5}$$

$$M_{\mathrm{TSV,sr}} = 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} M_{p1}(l_{\mathrm{TSV}}, d_{ij}) \tag{6}$$

$$M_{\mathrm{TSV,dr}} = \sum_{i=1}^{N} \sum_{j=1}^{N} M_{p1}(l_{\mathrm{TSV}}, d_{ij}) \tag{7}$$

$$M_{\mathrm{RDL},b} = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} M_{p1}(l_{\mathrm{RDL},b}, d_{ij}). \tag{8}$$

In a similar way, the total mutual inductance between TSVs from different rows can be calculated by (7), where $d_{ij} = (P_{\mathrm{TSV}}^2 + ((i-j)S_{\mathrm{TSV}})^2)^{1/2}$ is the distance between the ith and jth TSVs from different rows.

For simplicity, the RDL is simplified as a cylinder when calculating the mutual inductance in some works [4]. Therefore, the total mutual inductance between bottom-RDLs can be calculated by (8). Where $d_{ij} = (j-i)S_{\mathrm{TSV}}$ is the distance between the ith and jth bottom-RDL.

Fig. 1(b) shows the top view of the top-RDL in a 3-D inductor, as shown in Fig. 1(a). The mutual inductance between every two top-RDLs can be calculated by the formula of (9) and (10) [6]. Where l and m are lengths of two RDLs, respectively. The two RDLs have a center-to-center separation of d, and their endpoints are offset by a distance p. These parameters are all defined in Fig. 1(b).

The two IO-RDLs have a center-to-center distance of 0 and an offset of $p_N = NS_{\mathrm{TSV}}$, which are approximately perpendicular to other RDLs, so the mutual inductance between IO-RDL and other RDLs is extremely small and can be ignored. The total mutual inductance between top-RDLs can be derived by (11), where i and j denote the ith and jth top-RDLs. For example, Fig. 1(b) shows the geometries of the second and fourth top-RDLs. l_{ij} and m_{ij} are lengths of two RDLs, here excepts $l_{1j} = m_{iN+1} = l_{\mathrm{RDL},t}/2$, other $l_{ij} = l_{\mathrm{RDL},t}$. The offset p and center-to-center distance d of the ith and jth top-RDLs can be calculated by (12) and (13).

Fig. 2 shows the view of two RDLs from different layers. The mutual inductance between these two RDLs can be derived by (14) [7], where $d = l_{\mathrm{TSV}} + t_{\mathrm{RDL}}$ is the distance between the top and bottom layers, l and m denote the length of two RDLs, R_1, R_2, R_3, and R_4 are distances between every two endpoints of top-RDL and bottom-RDL, which are all defined in Fig. 2(a). The angle θ is identical between every two top-RDLs and bottom-RDLs, as shown in Fig. 2(b), which can be determined by the trigonometric function of P_{TSV} and S_{TSV}. The distance α and β between third top-RDL and first bottom-RDL are defined in Fig. 2(b). Ω is the function of d, α, β, θ, R_1, R_2, R_3, and R_4. So the total mutual inductance between RDLs from different layers can be derived by (15).

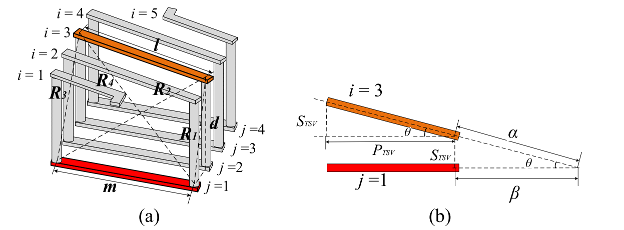

Fig. 2. (a) Perspective view. (b) Top view of two RDLs from different layers.

The distance α and β between the ith top-RDL and jth bottom-RDL can be determined by (16)

$$M(l,d) = \frac{\mu}{2\pi}\left(l \sinh^{-1}\left(\frac{l}{d}\right) - \sqrt{l^2+d^2} + d \right) \tag{9}$$

$$\begin{aligned} M_{p2}(l,m,p,d) &= 0.5(M(p+m,d) + M(p-l,d) \\ &\quad - M(m+p-l,d) - M(p,d)) \end{aligned} \tag{10}$$

$$\begin{aligned} M_{\mathrm{RDL},t} &= \sum_{i=1}^{N} \sum_{j=i+1}^{N+1} M_{p2}(l_{ij}, m_{ij}, p_{ij}, d_{ij}) \\ &\quad + M_{p2}(l_{\mathrm{RDL,io}}, l_{\mathrm{RDL,io}}, p_N, 0) \end{aligned} \tag{11}$$

$$p_{ij} = (j-i)\cdot S_{\mathrm{TSV}} \sin\theta = (j-i)\cdot \frac{S_{\mathrm{TSV}}^2}{\sqrt{P_{\mathrm{TSV}}^2 + S_{\mathrm{TSV}}^2}} \tag{12}$$

$$d_{ij} = (j-i)\cdot S_{\mathrm{TSV}} \cos\theta = (j-i)\cdot \frac{S_{\mathrm{TSV}} P_{\mathrm{TSV}}}{\sqrt{P_{\mathrm{TSV}}^2 + S_{\mathrm{TSV}}^2}} \tag{13}$$

$$\begin{aligned} M_{p3}&(l,m,d,\alpha,\beta,\theta) \\ &= \frac{\mu}{2\pi}\cos\theta \Bigg[(\alpha+l)\tanh^{-1}\frac{m}{R_1+R_2} + (\beta+m) \\ &\qquad \times \tanh^{-1}\frac{l}{R_1+R_4} - \alpha \tanh^{-1}\frac{m}{R_3+R_4} \\ &\qquad - \beta \tanh^{-1}\frac{l}{R_2+R_3} \Bigg] - \frac{\mu}{4\pi}\frac{\Omega d}{\tan\theta} \end{aligned} \tag{14}$$

$$\begin{aligned} M_{\mathrm{RDL,diff}} &= \sum_{i=1,N+1} \sum_{j=1}^{N} M_{p3}(l_{\mathrm{RDL},t}/2, l_{\mathrm{RDL},b}, d, \alpha_{ij}, \beta_{ij}, \theta) \\ &\quad + \sum_{i=2}^{N} \left(\sum_{j=1}^{i-1} + \sum_{j=1}^{N+1-i} \right) \\ &\quad \times M_{p3}(l_{\mathrm{RDL},t}, l_{\mathrm{RDL},b}, d, \alpha_{ij}, \beta_{ij}, \theta) \end{aligned} \tag{15}$$

$$\alpha_{ij} = \frac{(j-1)S_{\mathrm{TSV}}}{\sin\theta}, \quad \beta_{ij} = \frac{(j-1)S_{\mathrm{TSV}}}{\tan\theta} \tag{16}$$

$$\begin{aligned} L_{\mathrm{tot}} &= L_{\mathrm{TSV},S} + L_{\mathrm{RDL},S} \\ &\quad + 2(M_{\mathrm{TSV,sr}} - M_{\mathrm{TSV,dr}} + M_{\mathrm{RDL},b} \\ &\qquad + M_{\mathrm{RDL},t} - M_{\mathrm{RDL,diff}}). \end{aligned} \tag{17}$$

Note that the currents flowing in the TSVs are orthogonal to that in the RDLs, thus the mutual–partial inductance between TSVs and RDLs is zero. The mutual inductance between TSVs from the same row is positive due to the currents with the same direction flowing through them. Conversely, the mutual inductance between TSVs from different rows is negative. Similarly, the RDLs from the top or bottom layer contribute the positive mutual inductance, while the RDLs from different layers contribute to the negative mutual inductance. In addition, every segment in 3-D inductor is connected in series, the total inductance of N-turns inductor can be derived by (17), which

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TABLE I

COMPARISON OF INDUCTANCE AND SIMULATION TIME FROM Q3D AND MODEL

	Inductance (nH)			Percent error (%)		Simulation time (sec)		
N	Q3D	Model [4]	Our Model	Model [4]	Our Model	Q3D	Model [4]	Our Model
1	0.60	0.53	0.62	11.67	3.33	75	0.014	0.020
2	1.69	1.55	1.70	8.28	0.59	102	0.019	0.028
3	3.05	2.88	3.04	5.57	0.33	172	0.021	0.032
4	4.59	4.43	4.56	3.49	0.65	205	0.025	0.045
5	6.24	6.14	6.18	1.60	0.96	268	0.027	0.049
6	7.97	7.85	7.88	1.51	1.12	286	0.030	0.052
7	9.75	9.92	9.64	1.75	1.13	314	0.032	0.053
8	11.58	11.95	11.43	3.2	1.25	335	0.033	0.055
9	13.46	14.06	13.25	4.46	1.56	365	0.035	0.056
10	15.34	16.22	15.09	5.74	1.63	412	0.038	0.058

is the sum of the self-inductance of every segment and mutual inductance between every two segments.

III. VERIFICATION AND DISCUSSION

In this section, the accuracy of the proposed model is verified by the ANSYS Q3D extractor. Here the simulation model has l_{TSV} of 200 μm, r_{TSV} of 10 μm, w_{RDL} of 20 μm, t_{RDL} of 4 μm, P_{TSV} of 300 μm, and S_{TSV} of 40 μm.

Table I shows the comparison of inductance results and simulation time between Q3D and models that are proposed in [4] and this letter. It can be found that for the inductor with turns from 1 to 10, our model has a higher accuracy and a smaller error fluctuation than the model from [4], the error is less than 3.5%. The reason is that the internal inductance is considered in the calculation of self-inductance and the case of complicated RDL layers with offset and at different planes are considered in the calculation of mutual inductance. Compared with the Q3D extractor, the simulation time is reduced by a large factor using the analytical model from [4] and this letter. For the higher precision, more complex formulas are used in our model, so the simulation time is slightly larger than the model from [4], but there is no effect on the design of 3-D inductor.

As shown in Table I, the error trend of our model is fluctuant. We use terms of a calculation error and method error to explain the fluctuation. The calculation error can be determined by (18), where$\sum L_{\mathrm{partial_simulation}}$ is the sum of the partial inductance of every segment in 3-D inductor obtained by simulation, $\sum L_{\mathrm{partial\_calculation}}$ is the sum of the partial inductance of every segment obtained by calculation. So the error is introduced by the formulas of partial self and mutual inductance of TSV and RDL, which is increasing as the number of turns increase. The method error can be determined by (19), where $L_{\mathrm{total\_simulation}}$ is the total inductance of the 3-D inductor obtained by simulation. The error is introduced by the magnetic variation at the junction of TSV and RDL, it brings about some deviations between the summation of partial inductance and full inductance. The variation of calculation error and method error with turn number (N) is shown in Fig. 3. These two errors have opposite signs and can cancel each other out. So the error trend of our model is first decrease and then increase

$$\text{Calculation error} = \frac{\sum(L_{\mathrm{partial\_simulation}}) - \sum(L_{\mathrm{partial\_calculation}})}{\sum L_{\mathrm{partial\_simulation}}} \quad (18)$$

$$\text{Method error} = \frac{L_{\mathrm{total\_simulation}} - \sum(L_{\mathrm{partial\_calculation}})}{L_{\mathrm{total\_simulation}}}. \quad (19)$$

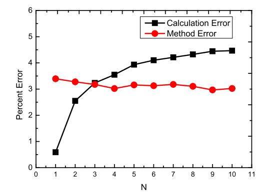

Fig. 3. Variation of calculation error and method errors with N.

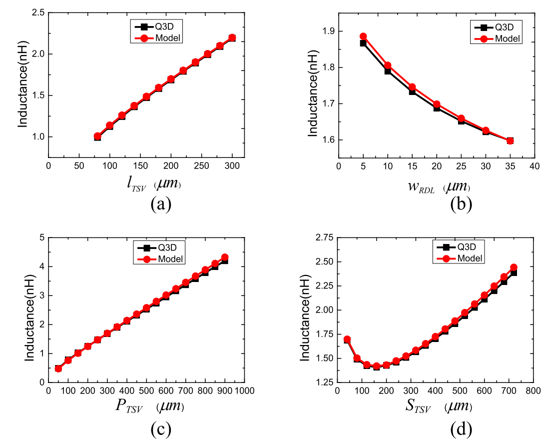

Fig. 4. Comparison of inductance between model and Q3D with various design parameters. (a) l_{TSV}. (b) w_{RDL}. (c) P_{TSV}. (d) $S_{\mathrm{TSV}}(N = 2)$.

Fig. 4 compares the analytical and simulation results of total inductance with various l_{TSV}, w_{RDL}, P_{TSV}, and S_{TSV}. The comparison shows a good agreement over various design parameters ranges, which further verified the accuracy of the model.

IV. CONCLUSION

An analytical model to accurately calculate the total inductance of 3-D inductor based on TSV is proposed in this letter, which provided results in good agreement with the simulation results by Q3D but uses negligible time. When the simulation and analytical results are compared for the inductance of the inductor with turns from 1 to 10, the error is within 3.5%. So our model can be used as a simple and precise method to estimate the inductance of the 3-D inductors. Finally, the reason for the error variation of our model is discussed.

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