High-Speed Channel Equalization Applying Parallel Bayesian Machine Learning

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***Abstract*—Recovering attenuated signals caused by different issues including connections between connectors and chips of the devices, channel loss, and crosstalk is a challenging problem. Equalization is the most popular way to restore distorted signals. Different optimization algorithms are used to find the best tap coefficients for each equalization that improves eye opening and decreases the bit error rate (BER). Nowadays algorithms of equalization mainly operate to reduce the difference between input and output signals. In turn, this will increase eye height indirectly, but direct maximization of the eye height will restore the signal even better. The paper proposes a new efficient optimization of joint Feed-Forward Equalization (FFE) and Decision Feedback Equalization (DFE) for binary as well as multi-level signals. Unlike to above-mentioned method, which is a linear optimization problem, direct maximization of the eye height applying joint FFE and DFE is a non-linear problem and cannot be solved analytically. Paper proposes the black-box function optimization using the regular as well as parallel Bayesian machine learning to find the best tap coefficients for joint FFE and DFE equalization. The efficiency of the parallel Bayesian algorithm is shown for binary NRZ (nonreturn-to-zero) and PAM4 (Pulse Amplitude Modulation) signals.**

Keywords—Equalization; DFE; FFE; Optimal selection of tap coefficients; Joint optimization; Bayesian Optimization; Parallel algorithm

I. Introduction

One of the important problems of electrical engineering nowadays is increasing signal transmission speed, which is directly related to increasing data transmission and processing speed in electronic devices. In addition, it is important to keep or decrease material cost as well as sizes of microschemes, chips and other electronic components. In modern electronic devices and networks, such as computers and networking equipment, data is transmitted using a digital signal. In general, the signal represents an electrical wave that encodes the data in a binary way by voltage change. Nowadays the data mainly is transmitted by a two-level signal that is an electrical wave that represents a sequence of two different levels of voltages. The receiving component decodes this signal. The electronic devices consist of many different PCBs, chips, connectors, and components. The connection between each component distorts the signal. It is not a big problem in low-speed devices, but modern technologies constantly increase the signal speed along with the development of smartphones, computational centers and in general computers. Current data rates exceed 10 Gbps, and next generation technologies will be 56 Gbps, and in 2020 the IEEE latest standard predicts 200 Gbps. Such increasing of signal speed results in new problems. Signals traveling from transmitter to receiver are distorted due to different factors such as connections between device chips and connectors, channel length loss, crosstalk between parallel channels, inter-symbol interference, jitter, and noise. Consequently, an important part of Signal Integrity (SI) problems is devoted to signal restoration. Equalization is the most popular way to restore the compromised signal. There exist different types of equalization, though generally, they represent the scheme of signal correction. The most popular equalizers are Continuous-Time Linear Equalization (CTLE), Linear Feedforward Equalization (LFE, also known in the literature as FFE), and Decision Feedback Equalization (DFE) [1] - [9]. The cost function for common equalization optimization algorithms is to lower the difference between the loaded (training) and received (distorted) signals, which is a linear optimization problem and can be solved relatively easily by applying analytical methods. This indirectly will increase the height of the eye diagram. Direct optimization of eye height by using the joint FFE and DFE equalization is a non-linear problem and cannot be solved analytically. For nonreturn-to-zero (NRZ) type signals with PRBS-15 (pseudo-random binary sequence) bit sequence, combined FFE and DFE equalization is proposed in [3]. In [4], the goal function minimizes Signal to Noise Ratio (SNR) based on the

This paper is based upon work supported partially by the National Science Foundation under Grant No. IIP-1916535.

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collaboration of FFE and DFE tap coefficients. The proposed optimization problem is linear regarding FFE and DFE tap coefficients and is solved analytically. Likewise to [3], the study in [4] is done for NRZ.

In the presented paper, the equalization optimization algorithm is applied for 4 level Pulse Amplitude Modulation (PAM4) signal with Quaternary PRBS-13 (QPRBS-13) bit sequence. We are proposing FFE and DFE combined equalization optimization algorithm that directly optimizes eye height using Bayesian Optimization (BO). The wide-spread approach is to define the specific intervals for each tap coefficient and then sweep through these intervals with predefined step, which is a very ineffective and time-consuming approach. By combining the iterative methods and Bayesian machine learning, in this study, the BO algorithm chooses optimal parameters for combined equalization method in real-time. The application of Bayesian optimization is one of the popular approaches for many engineering problems, especially dealing with multi-dimensional cases (see, for example, [10], [11]). Unlike to gradient-based optimization methods, BO does not require information about derivatives. Consequently, it can be used as a powerful derivative-free optimization method for black-box functions. BO is based on Gaussian Process (GP) and Bayes Theorem (for detail see, [12] - [14]). We are also proposing a parallel BO for finding the best values of FFE and DFE tap coefficients for signal equalization. Obtained results, in both cases for NRZ and PAM4, show that parallel BO saves computational time comparing to the regular BO [3].

The paper is organized as follows. In Section II, some mathematical background for FFE and DFE equalization as well as for BO is given. Both sequential and parallel algorithms are described too in this section. In Section III, parallel BO is applied for Peaks function and for joint FFE/DFE equalization for NRZ (with PRBS-15) and PAM4 (with QPRBS-13) signals. The results of numerical experiments are done in the same section. The conclusions are drawn in Section IV finally.

II. Some Mathematical Background

A. FFE and DFE Optimization Problem

Let us consider FFE equalization of a system at first. Assume $u_k, k = 1,2,\dots,N$ is an input signal and $v_k, k = 1,2,\dots,N$ is a corresponding output. FFE equalized signal is computed according to the following equation:

$$v_k^{FFE} = \sum_{i=-m}^{n} c_i v_{k-i}\,, \tag{1}$$

where $c_i, i = -m,\dots,n$, are FFE tap coefficients. Negative indices correspond to pre-cursor tap coefficients; positive indices correspond to post-cursor tap coefficients and c_0, is a cursor value (Fig. 1).

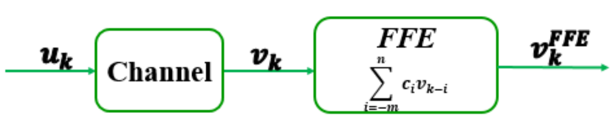


Fig 1. Linear Feedforward Equalization

After FFE equalization, for more improvement of the attenuated signal, DFE equalization can be applied. Taking into account (1), with known DFE tap coefficients DFE equalization is given by the following equation:

$$v_k^{FFE/DFE} = v_k^{FFE} - \sum_{i=1}^{p} d_i sign(v_{k-i}^{FFE})\,, \tag{2}$$

where $d_i,\ i = 1,\dots,p$, are DFE tap coefficients. In (2), function $sign(x)$ is a discontinuous, non-linear function which equals to 1 for positive x and -1 for negative x. Flowchart for combined FFE/DFE equalization is given in Fig. 2.

Fig 2. Combined FFE/DFE equalization scheme

The purpose of the FFE/DFE optimization is to find the best values for tap coefficients to maximize difference between lowest logical 1 and highest logical 0 for NRZ and analogically to maximize difference between corresponding logical levels for PAM4. In figures below, levels for NRZ are 1 and -1, whereas levels for PAM4 are -1, -1/3, 1/3, and 1.

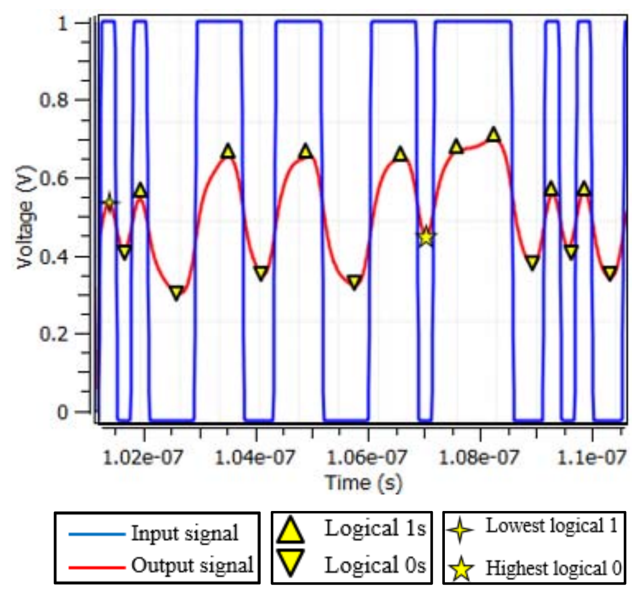


Fig 3. Logical levels for NRZ

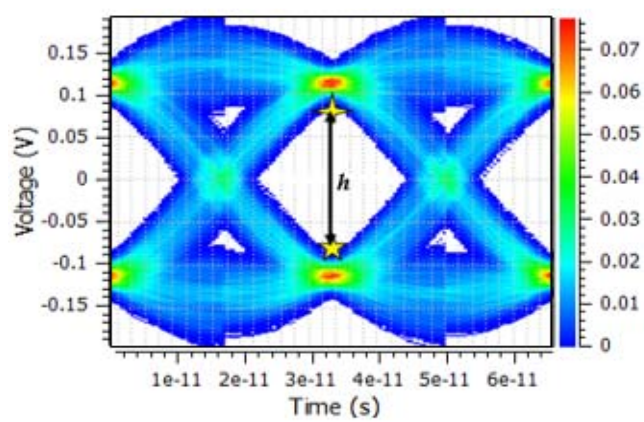


Fig 4. Eye diagram for NRZ

In Fig. 3 the portion of input and output signals are given. Considering each Unit Interval (UI) of whole signals, all logical 0 and 1 for the output signal are compared and the lowest logical 1 ($Logic_1$) and highest logical 0 ($Logic_0$) are found. Fig. 4 shows an eye diagram with eye height:

$$h_{NRZ} = Logic_1 - Logic_0. \tag{3}$$

For PAM4 the differences between logical levels are defined for top, middle, and bottom eyes (Fig. 5).

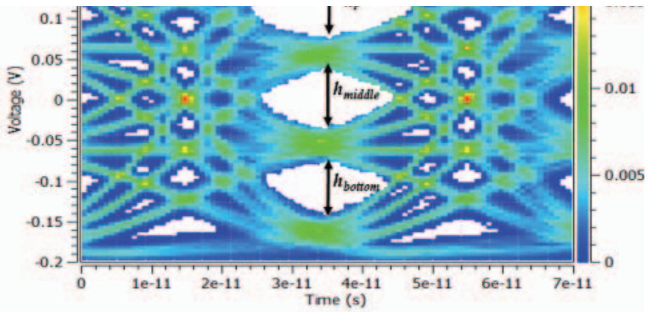


Fig 5. Eye diagram for PAM4

Eye heights for PAM4 are calculated similarly as for NRZ. They represent differences between corresponding logic levels for each UI:

$$\begin{aligned} h_{top} &= Logic_1 - Logic_{2/3}, \\ h_{middle} &= Logic_{2/3} - Logic_{1/3}, \\ h_{bottom} &= Logic_{1/3} - Logic_0. \end{aligned} \tag{4}$$

Cost function incorporates FFE and DFE tap coefficients together and from equalities (4) constructs the output value as follows:

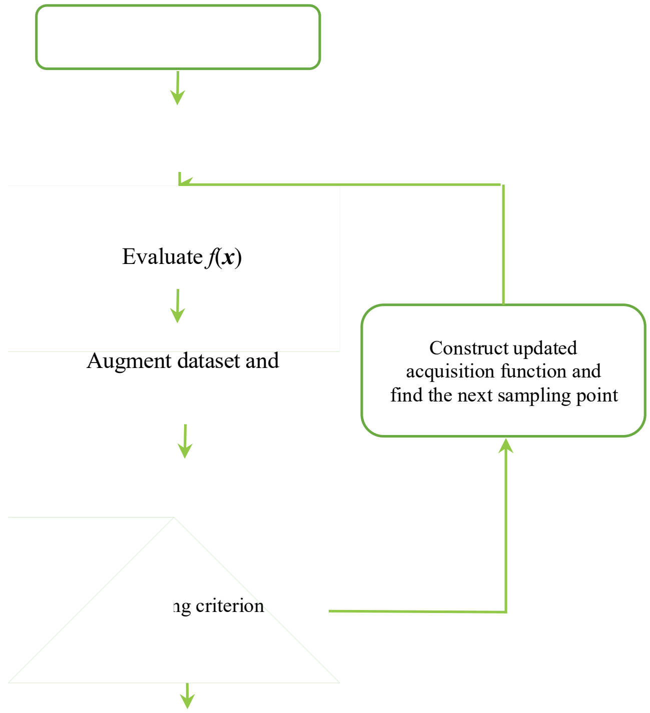
$$h_{PAM4} = \min(h_{top}, h_{middle}, h_{bottom}). \tag{5}$$

As we see from (3) and (5), there is no analytical representation for h_{NRZ} and h_{PAM4} showing explicit dependence on FFE and DFE tap coefficients. Therefore, to maximize eye heights the black-box function optimization method is needed. We applied BO for the mentioned black-box optimization and parallelized process to speed up the convergence. Description for parallelization and a brief overview of general Bayesian optimization is given in the next subsection.

B. A Brief Description of the Algorithm

As we noted in the introduction, the Bayesian Optimization is based on Gaussian Process and Bayes Theorem. The Gaussian Process is an extension of Multivariate Gaussian Distribution. In turn, the multivariate Gaussian distribution is a generalization of one-dimensional Gaussian distribution to higher dimensions. Whereas a Gaussian probability distribution describes random variables Gaussian process governs the properties of functions [12] - [14]. One can imagine of a function as a very long vector, each entry in the vector specifying the function value at a particular input In this study, we avoid the description of GP and BO, though we focus on the parallelization of the BO. For details regarding GP and BO, we refer the interested reader to [3], [10] - [16].

The main steps for the regular BO algorithm is given on the flowchart bellow (Fig. 6) [11]:



he regular BO algorithm

Since BO can start from any point from the searching space [16], in order to parallelize BO, let us run several BO with different random initial guesses. These several BOs should run for fewer iterations (small BOs) comparing to running one regular BO. After all small BOs finish, the best solution is taken and that solution is passed as an initial guess to the new BO (the regular one). The flowchart for parallel BO is given in Fig. 7.

On the third step, the algorithm for regular BO (shown on Fig. 6) is used for each BO_i. For implementation of all steps in parallel BO, we slightly modified IMGPO (Infinite-Metric GP Optimization) and used it for the third step (for details regarding IMGPO see [11] and [16]).

III. Results of Numerical Experiments

Before applying parallel BO to the FFE and DFE joint equalization, without the loss of generality, let us verify its efficiency for the well-know mathematical function such as Peaks function. Equation for Peaks function is given as follows:

$$f(x,y) = 3(1-x)^2 e^{-x^2-(y+1)^2}$$
$$-10\left(\frac{x}{5} - x^3 - y^5\right) e^{-x^2-y^2} - \frac{1}{3} e^{-(x+1)^2-y^2},$$

where $(x, y) \in [-3; 3] \times [-3; 3]$. To find the maximum, the brute force search gave the following result: $x = -0.0093$, $y = 1.58136$, and $f(x, y) = 8.106213585827$. The graphical illustration for Peaks function (6) is given in Fig. 8.

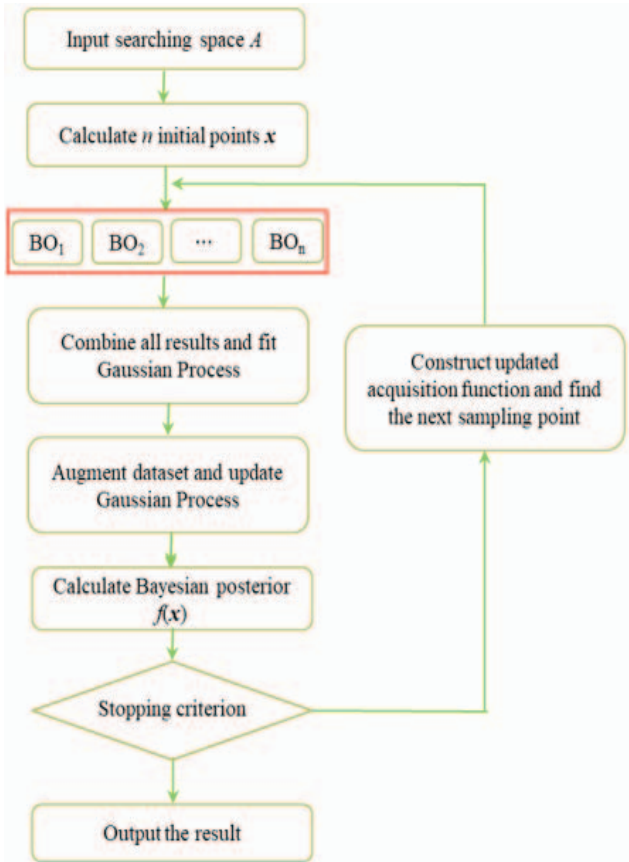


Fig 7. A flowchart for the parallel BO algorithm

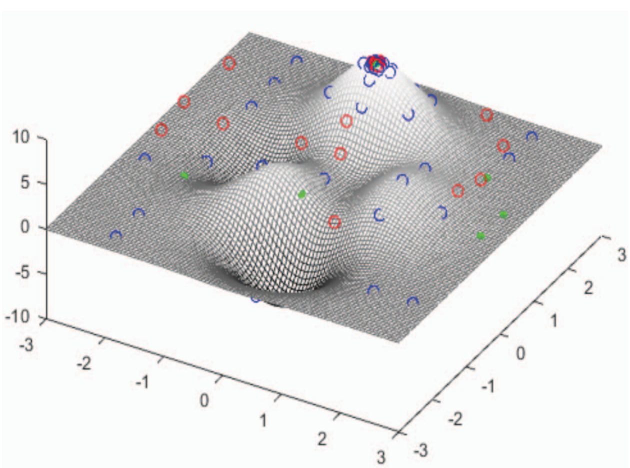


Fig 8. Convergence to the global maximum for Peaks function

To compare the results of the regular and parallel BO algorithms, at first, regular BO for 1000 iterations and next, three parallel BOs were run for 300 iterations each with different initial guesses. The results of all three parallel BOs were combined and the best solution was found. After, BO was run once again with the already found the best solution as an initial guess. The obtained results are given in Table I.

TABLE I. RESULTS FOR REGULAR AND PARALLEL BO

	x	*y*	Max	Iterations	Minutes
Regular BO	-0.00932	1.581368	8.106214	1000	~2.8
Parallel BO	-0.00823	1.5802	8.1062	300 in parallel + 300 after	~0.78 in total

As we see from Table I, the parallel BO gives the almost same result (with an accuracy of 10^{-3}) in much less computational time (about 3.6 faster). We tried parallel BO for other multi-variable mathematical functions and observed the effectiveness of parallel BO.

In [3] a high speed channel is considered to compare BO based equalization against Least Mean Square (LMS) equalization. Eye opening is compared for NRZ signal with 50 Gbps data rate. Fig. 9 shows insertion and return losses of the channel. The channel comprises of 5” transmission line on a board with Megtron-6 material (ultra-low loss material) connecting to another board with 13” of transmission line on Megtron-6. There are total of 4 vias in the path with each via having less than 10 mils of via stub. The connector used for connecting the boards is a high frequency connector designed for 50 Gbps speeds [3].

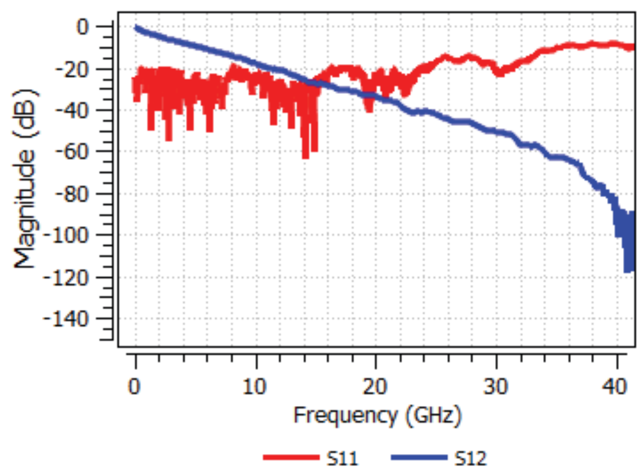


Fig. 9. Insertion loss and return loss of the test channel

When the data rate is 50 Gbps, the comparison of BO versus LMS equalization for the NRZ signal shows that BO gives about 70% better results in eye opening [3]. Now let us study how the direct maximization of the eye heights for NRZ as well as for PAM4 signals with 56 Gbps data rate improves the results versus LMS. Let us use the same channel, 3 tab coefficients for both FFE and DFE, and the regular BO with 500 iterations.

The results for LMS and joint FFE/DFE equalization with regular BO are shown in Figs. 10 and 11 for NRZ and in Figs. 12 and 13 for PAM4 respectively. The corresponding approximate values for eye heights and widths are given in Tables II and III.

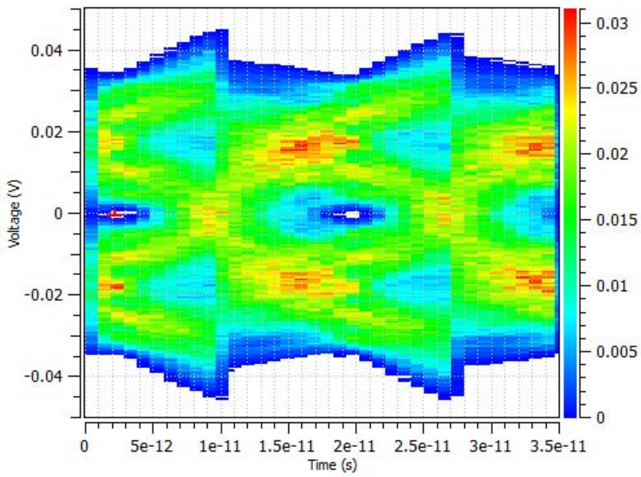


Fig. 10. Eye diagram of separate FFE/DFE equalized channel response using LMS for NRZ signal

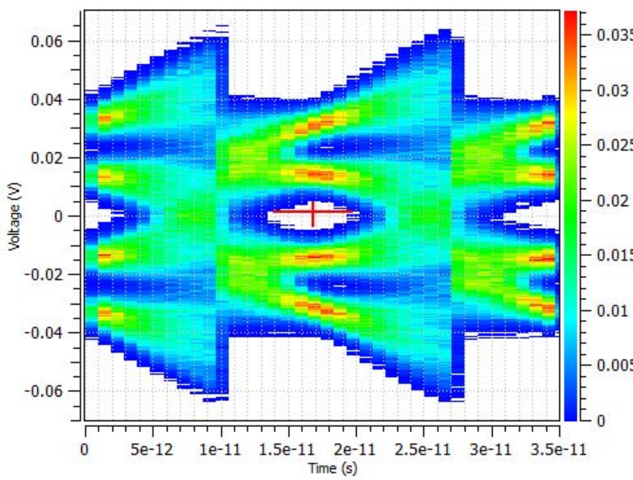


Fig. 11. Eye diagram of joint FFE/DFE equalized channel response using regular BO for NRZ signal

TABLE II. RESULTS FOR LMS AND BO (NRZ)

Eye	Least Square	Joint FFE/DFE with BO
Eye height	1.21824 mV	8.88067 mV
Eye width	1.21984 ps	5.88222 ps

TABLE III. RESULTS FOR LMS AND BO (PAM4)

Eye	Least Square	Joint FFE/DFE with BO
Top eye height	15.0156 mV	23.9361 mV
Top eye width	6.42458 ps	9.31958 ps
Middle eye height	14.8003 mV	24.0512 mV
Middle eye width	7.03863 ps	9.57853 ps
Bottom eye height	13.72 mV	23.9575 mV
bottom eye width	6.58227 ps	8.15993 ps

As we see, for the NRZ signal, LMS almost could not open the eye, whereas BO did and consequently, based on data shown in Table II, gave about 7 times better result for eye height and approximately 5 times better result for eye width. According to data in Table III, in the PAM4 case, eye height/width for joint optimization is more than 59%/45%, 62%/36%, and 74%/23% better for top, middle, and bottom eyes correspondingly.

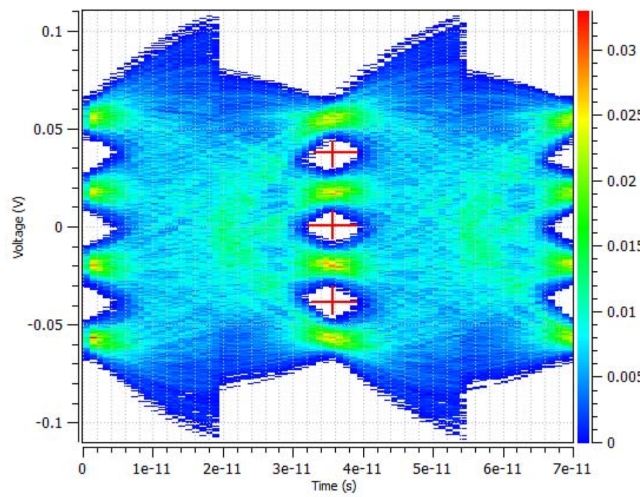


Fig. 12. Eye diagram of separate FFE/DFE equalized channel response using LMS for PAM4 signal

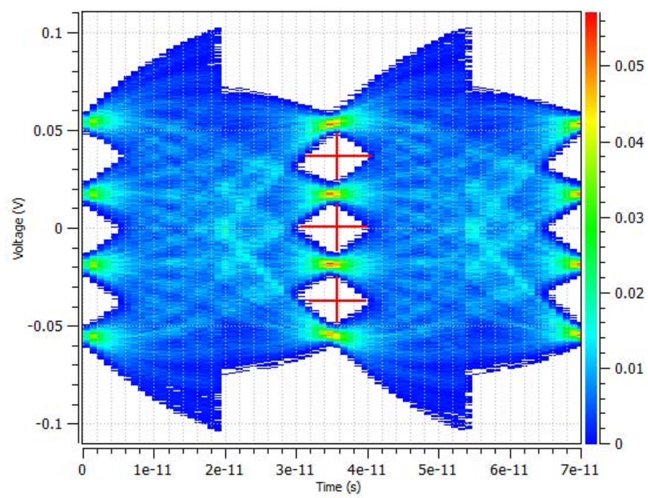


Fig. 13. Eye diagram of joint FFE/DFE equalized channel response using regular BO for PAM4 signal

Now let us study the parallel BO for both NRZ and PAM4 signals and investigate how the results can be improved in terms of computational time. To compare the results of the regular and parallel BO algorithms, let us run regular BO for 400 iterations at first, and next run three parallel BOs for 100 iterations each with different (uniformly distributed on the searching space) initial guesses, combine the results of all three BOs and determine the best solution. Finally, run one more BO for 300 iterations with the already found best solution as an initial guess. The above-mentioned procedure is applied for NRZ as well as for PAM4. The results are given in Tables IV and V respectively. As we see, parallel BO gives better results

42% faster for NRZ and 43% faster for PAM4. Note that as a searching space [-1; 1] and [-0.1; 0.1] segments are used for all FFE tap coefficients and for all DFE tap coefficients respectively.

TABLE IV. RESULTS FOR NRZ USING REGULAR AND PARALLEL BO

Eye	Regular BO	Time	Parallel BO	Time
Eye height	8.88067 mV	~12.4 minutes	9.53212 mV	~8.7 minutes
Eye width	5.88222 ps		7.12424 ps	

TABLE V. RESULTS FOR PAM4 USING REGULAR AND PARALLEL BO

Eye	Regular BO	Time	Parallel BO	Time
Top eye height	23.9361 mV	~11.9 minutes	23.9366 mV	~8.3 minutes
Top eye width	9.31958 ps		9.31961 ps	
Middle eye height	24.0512 mV		24.0516 mV	
Middle eye width	9.57853 ps		9.57853 ps	
Bottom eye height	23.9575 mV		23.9573 mV	
bottom eye width	8.15993 ps		8.15995 ps	

IV. CONCLUSIONS

The paper proposes a combined FFE and DFE optimization algorithms for NRZ and PAM4 signals that directly optimize eye height. The goal of the considered algorithms is to maximize the difference between channel response values corresponding to the lowest and highest logical levels. To find the best values for FFE and DFE tap coefficients the non-linear black-box function optimization is done. Regular as well as parallel Bayesian Optimization algorithm is used to solve the mentioned non-linear problem. Proposed algorithms are used to optimize a high speed channel for 56 Gbps signals. Comparison analysis of the obtained results versus FFE and DFE Least Mean Square optimization is given. Application of parallel BO showed that more than 42% of the computational time could be saved without losing accuracy.

Acknowledgment. This work was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNFG), grant no. YS-19-2887.

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